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THE EFFECT OF ANGLE OF BEND BETWEEN PLATE ELEMENTS ON THE LOCAL INSTABILITY OF FORMED Z-SECTIONS

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THE EFFECT OF ANGLE OF BEND BETWEEN PLATE ELEMENTS ON THE LOCAL INSTABILITY OF FORMED Z-SECTIONS

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SUMMARY

Thirty-nine Z-sections formed from 24S-T aluminum-alloy sheet with the angle of bend between the flanges and the web varying from 5° to 120° were tested as columns. Angles of bend from 30° to 120° had little or no effect on the critical stress or on the average stress at maximum load for local instability of the columns. The lengths of the columns were such that at angles of bend below 30° they failed by Euler buckling.

INTRODUCTION

The theory for local instability of columns composed of plate elements (for example, see references 1 and 2) involves the assumption that, when local buckling occurs, the joints or corners between plate elements remain in their original straight lines. Experimental investigation (reference 3) indicates that this assumption is justified when the plate elements meet at angles of approximately 90°; when the angle between plates is considerably different from 90°, however, the applicability of the theoretical methods might reasonably be open to question.

In order to investigate the effect of a variation in the angle between plate elements of a column on the critical (buckling) stress and on the maximum (failure) stress for local instability, tests were made of Z-section columns of 24S-T aluminum alloy in which the angle of bend for the flanges was varied from 5° to 120°.

SYMBOLS

E	modulus of elasticity, ksi
σ_{cy}	compressive yield stress, ksi
b_W	width of web of Z-section, inches (see fig. 2)
b_F	width of flange of Z-section, inches (see fig. 2)
b_W'	web width adjusted to width for Z-section formed with elements joined at 90°, inches (see fig. 7)
b_F'	flange width adjusted to width for Z-section formed with elements joined at 90°, inches (see fig. 7)
b_o	developed width from edge of sheet to middle of curved portion, inches (see fig. 7)
b_r	developed width from edge of sheet to beginning of curved portion, inches (see fig. 7)
t	thickness of 24S-T aluminum-alloy sheet, inches
r	radius of bend, inches (see fig. 2)
θ	angle of bend, degrees (see fig. 2)
L	length of Z-section column, inches
ρ	radius of gyration for Z-section, inches

SPECIMENS

All specimens were formed from the same sheet of 0.081-inch 24S-T aluminum alloy with the grain of the material parallel to the length of the specimens. Compressive stress-strain curves for the 24S-T sheet are shown in figure 1. Typical test specimens and the measured dimensions are shown in figure 2. The same developed width of flat sheet was used to make all columns. The columns were so formed that the developed width from the edge of the sheet to the middle of the curved portion was constant. A median-line radius of bend of four times the nominal sheet thickness was used.

In order to avoid Euler failure and to cover as wide a range of local instability as possible, a nominal ratio of flange width to web width b_F/b_W of 0.6 was used. The angle of bend between web and flange was varied from 5° to 120°. For each angle of bend three specimens were made; two with a ratio of column length to web width L/b_W equal to 4 and the third with this ratio equal to 5. The measured dimensions of the specimens are given in table I.

METHOD OF TESTING

The column tests were made in a 300,000-pound-capacity compression testing machine that is accurate within 1 percent for the range of loads used in these tests.

The displacement of pointers, supported by extension arms attached to the flanges of the column (see fig. 3), was taken as a measure of the distortion of the cross section. Optical micrometers, as shown in figure 3, were used to determine this displacement. The critical compressive stress was obtained from stress-distortion curves in the manner outlined in reference 3. In this method the critical compressive stress is determined as the point near the top of the knee of the stress-distortion curve at which there first occurs a pronounced increase in distortion with small increase in stress.

TEST RESULTS

The stresses developed by the columns tested are listed in table I and are plotted against the angle of bend in figure 4. When the angle of bend is very small, the radius of gyration is also small, and the slenderness ratio L/ρ is large. The specimens with small angles of bend (30° or less) therefore developed Euler (bending) failures. For the specimens with larger angles of bend, the stress for Euler failure was higher than that for local instability, and the local-instability type of failure was therefore developed.

For those specimens that failed by local instability, figure 4 shows little effect of the angle of bend on either the critical stress or the average stress at maximum load.

The method of dimensioning of test specimens, illustrated in figure 2, gives web and flange widths that increase with increasing angle of bend, even though the nominal developed width of sheet remains constant. Figure 4 shows, however, that the critical and maximum stresses do not change greatly with changing angle of bend. In order to check the experimental results against calculated results, therefore, it is necessary to adjust the measured dimensions in some manner that will give comparable values for all specimens.

In the investigation of reference 3, all specimens were formed with 90° bends, and the dimensions were taken between intersections of center lines. This method of dimensioning provided a good check with theory. In obtaining the calculated values of critical and maximum stresses for the specimens of the present investigation, therefore, all measured dimensions were adjusted to values that the specimen would have had if it had been formed with 90° bends between plate elements. The method of making the adjustment is discussed in the appendix.

Figure 5 shows the comparison of experimental and calculated critical stresses. The calculated values were obtained from the charts of reference 1, with the measured dimensions adjusted as mentioned in the preceding paragraph. Figure 6 shows the comparison of experimental and calculated values of average stress at maximum load. The calculated values were based on the theoretical critical stresses and the extensive test results of reference 3, in which the angle of bend was 90°. Because the values are based on test results, the low points at $\theta = 90°$ in figure 6 simply indicate that those test points did not fall in line with the tests of reference 3. These low points, and probably also those at $\theta = 100°$, must therefore be ascribed to experimental scatter. Except for such experimental scatter, the points plotted in both figure 5 and figure 6 indicate a constant stress for all angles of bend included.

CONCLUSIONS

The angles of bend from 30° to 120° between plate elements had little or no effect on the critical stress or on the average stress at maximum load for local instability of Z-section columns. The lengths of the columns were such that at angles of bend below 30° the columns failed by Euler buckling.

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APPENDIX

METHOD OF ADJUSTING MEASURED DIMENSIONS

The measured dimensions shown in figure 2 were adjusted to the dimensions that the specimen would have had if it had been formed with 90° bends between plate elements. The method is indicated in figure 7, in which, for the general case,

$$b_F = b_r + r \tan \frac{\theta}{2}$$

However,

$$b_r = b_o - r \frac{\theta}{2}$$

so that

$$b_F = b_o - r\left(\frac{\theta}{2} - \tan \frac{\theta}{2}\right) \tag{A1}$$

or

$$b_o = b_F + r\left(\frac{\theta}{2} - \tan \frac{\theta}{2}\right) \tag{A2}$$

For the special case of $\theta = 90^\circ$ equation (A1) may be written

$$b_F = b_o - r\left(\frac{\pi}{4} - 1\right)$$

$$= b_o + 0.2146r \tag{A3}$$

If the adjustment of dimensions is made on the basis that the developed width from the edge of the sheet to the middle of the curved portion b_o is kept constant, as was nominally the case in the forming of the specimens, the evaluation of b_F' for any specimen may be made by substitution of equation (A2) into equation (A3); then

$$b_F' = b_F + r\left(0.2146 + \frac{\theta}{2} - \tan\frac{\theta}{2}\right) \qquad (A4)$$

Similarly,

$$b_W' = b_W + 2r\left(0.2146 + \frac{\theta}{2} - \tan\frac{\theta}{2}\right) \qquad (A5)$$

The values of b_F' and b_W' used in obtaining the calculated critical stress for the test specimens are listed in table I.

REFERENCES

1. Kroll, W. D., Fisher, Gordon P., and Heimerl, George J.: Charts for Calculation of the Critical Stress for Local Instability of Columns with I-, Z-, Channel, and Rectangular-Tube Section. NACA ARR No. 3K04, 1943.

2. Lundquist, Eugene E., Stowell, Elbridge Z., and Schuette, Evan H.: Principles of Moment Distribution Applied to Stability of Structures Composed of Bars or Plates. NACA ARR No. 3K06, 1943.

3. Heimerl, George J., and Roy, J. Albert: Preliminary Report on Tests of 24S-T Aluminum-Alloy Columns of Z-, Channel, and H-Section That Develop Local Instability. NACA RB No. 3J27, 1943.

TABLE I.- SPECIMEN DIMENSIONS AND TEST RESULTS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

Specimen	θ (deg)	b_W (in.)	b_F (in.)	t (in.)	r (in.)	L (in.)	Cross-sectional area (sq in.)	Critical stress (ksi)	Av. stress at max. load (ksi)	b_W' (in.)	b_F' (in.)
1-A	5.1	2.70	1.72	0.078	0.32	12.14	0.483	--------	[a]2.10	2.84	1.79
1-B	4.9	2.86	1.65	.079	.32	12.10	.487	--------	[a]2.30	3.00	1.72
1-C	5.5	2.78	1.67	.078	.32	16.10	.489	--------	[a]1.10	2.92	1.74
2-A	10.3	2.77	1.68	.078	.32	12.15	.483	--------	[a]5.60	2.91	1.75
2-B	10.4	2.74	1.70	.079	.32	12.11	.488	--------	[a]6.10	2.88	1.77
2-C	9.9	2.84	1.65	.078	.32	16.19	.491	--------	[a]2.50	2.98	1.72
3-A	20.6	2.79	1.67	.079	.32	12.12	.485	--------	[a]17.60	2.93	1.74
3-B	20.3	2.80	1.70	.079	.32	12.12	.487	--------	[a]16.40	2.94	1.79
3-C	20.3	2.76	1.69	.079	.32	16.09	.491	--------	[a]12.40	2.90	1.76
4-A	31.2	2.81	1.67	.079	.32	12.13	.489	17.38	[a]26.00	2.95	1.74
4-B	31.7	2.77	1.71	.079	.32	12.12	.493	15.80	[a]25.60	2.91	1.78
4-C	30.7	2.91	1.65	.079	.32	16.12	.491	16.09	[a]19.50	3.05	1.72
5-A	38.6	2.76	1.71	.078	.32	12.19	.483	15.90	26.50	2.88	1.77
5-B	40.2	2.78	1.70	.079	.32	12.20	.489	16.00	27.00	2.90	1.76
5-C	40.3	2.79	1.69	.079	.32	16.14	.490	16.20	25.41	2.91	1.75
6-A	50.4	2.80	1.70	.079	.32	12.16	.487	16.00	26.70	2.92	1.76
6-B	50.2	2.84	1.68	.079	.32	12.18	.486	15.80	26.70	2.96	1.74
6-C	50.2	2.86	1.67	.079	.32	16.05	.491	15.90	26.20	2.985	1.73
7-A	60.3	2.91	1.67	.079	.32	12.17	.485	16.80	27.10	3.01	1.72
7-B	60.1	2.81	1.71	.079	.32	12.11	.486	16.25	27.10	2.91	1.765
7-C	60.1	2.78	1.72	.079	.32	16.05	.491	15.70	26.80	2.88	1.77
8-A	70.3	2.99	1.67	.078	.32	12.15	.484	16.40	26.40	3.07	1.71
8-B	70.2	2.85	1.72	.079	.32	12.18	.487	15.40	27.10	2.935	1.76
8-C	70.1	2.83	1.73	.079	.32	16.10	.489	15.10	26.80	2.91	1.77
9-A	79.8	2.865	1.745	.079	.32	12.16	.488	15.30	26.50	2.905	1.765
9-B	79.9	2.875	1.74	.079	.32	12.16	.488	15.70	26.90	2.915	1.76
9-C	79.8	2.865	1.745	.079	.32	16.13	.491	15.90	26.60	2.905	1.765
10-A	89.6	2.915	1.77	.079	.33	12.12	.484	15.30	24.30	2.915	1.77
10-B	89.5	2.92	1.77	.078	.33	12.10	.489	15.75	24.00	2.92	1.77
10-C	90.6	2.94	1.755	.078	.32	16.05	.484	14.70	26.10	2.94	1.755
11-A	99.35	2.985	1.79	.079	.32	12.17	.486	15.20	24.80	2.925	1.76
11-B	99.7	2.97	1.81	.078	.32	12.14	.483	14.92	26.60	2.91	1.78
11-C	98.8	2.985	1.79	.078	.32	16.12	.488	15.80	23.30	2.925	1.76
12-A	109.5	3.125	1.85	.079	.32	12.15	.489	15.38	26.70	2.965	1.77
12-B	109.4	3.135	1.85	.079	.33	12.16	.486	15.40	27.40	2.975	1.77
12-C	109.4	3.15	1.84	.079	.33	16.00	.488	16.00	26.30	2.99	1.76
13-A	120.5	3.26	1.93	.079	.33	12.06	.489	16.15	27.00	2.942	1.771
13-B	119.7	3.25	1.915	.079	.31	12.10	.488	17.00	27.00	2.964	1.772
13-C	118.5	3.26	1.91	.078	.32	16.20	.483	16.80	25.40	2.984	1.772

[a]Column failure.

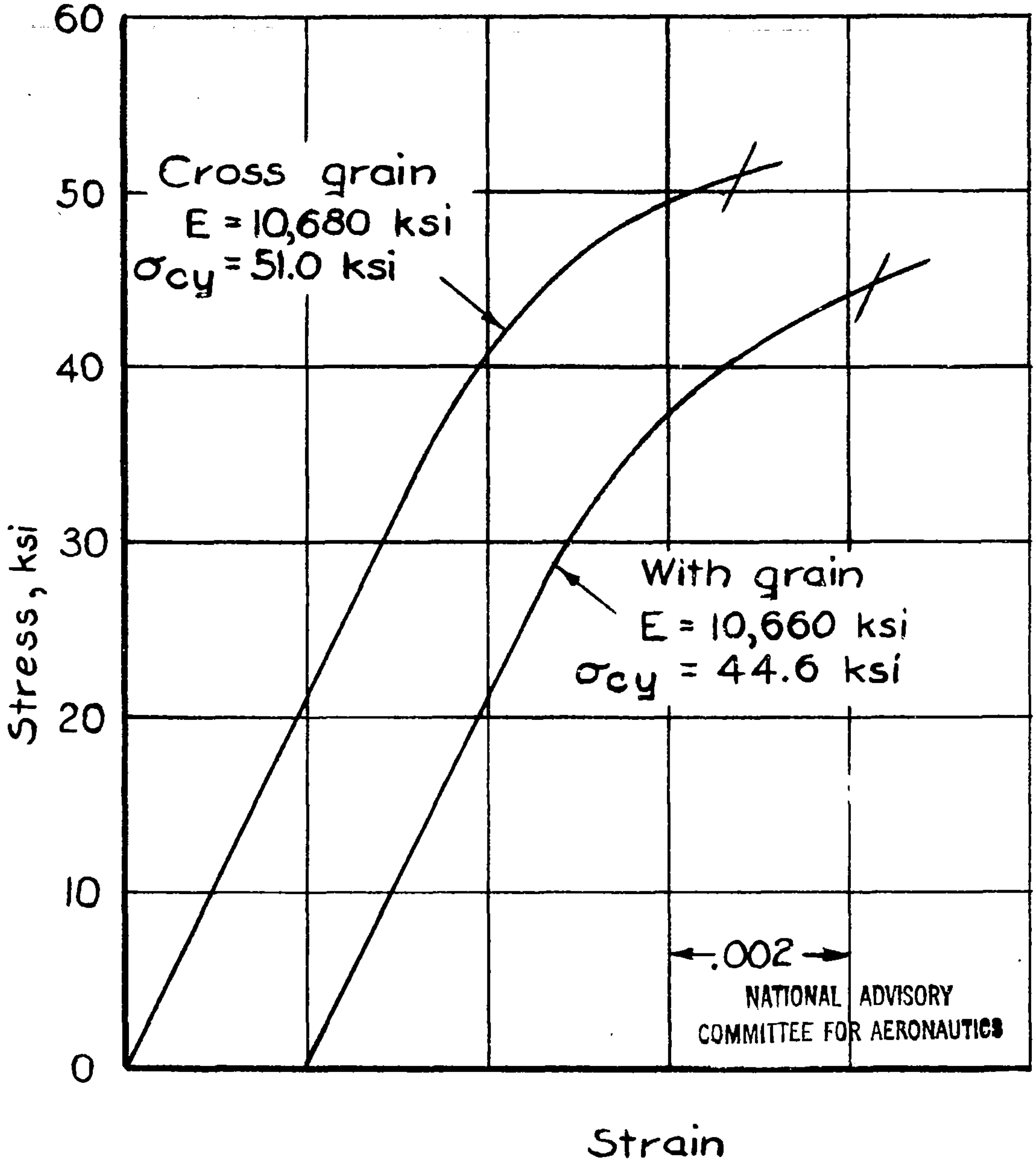

Figure 1.- Compressive stress-strain curves for 24 S-T aluminum alloy.

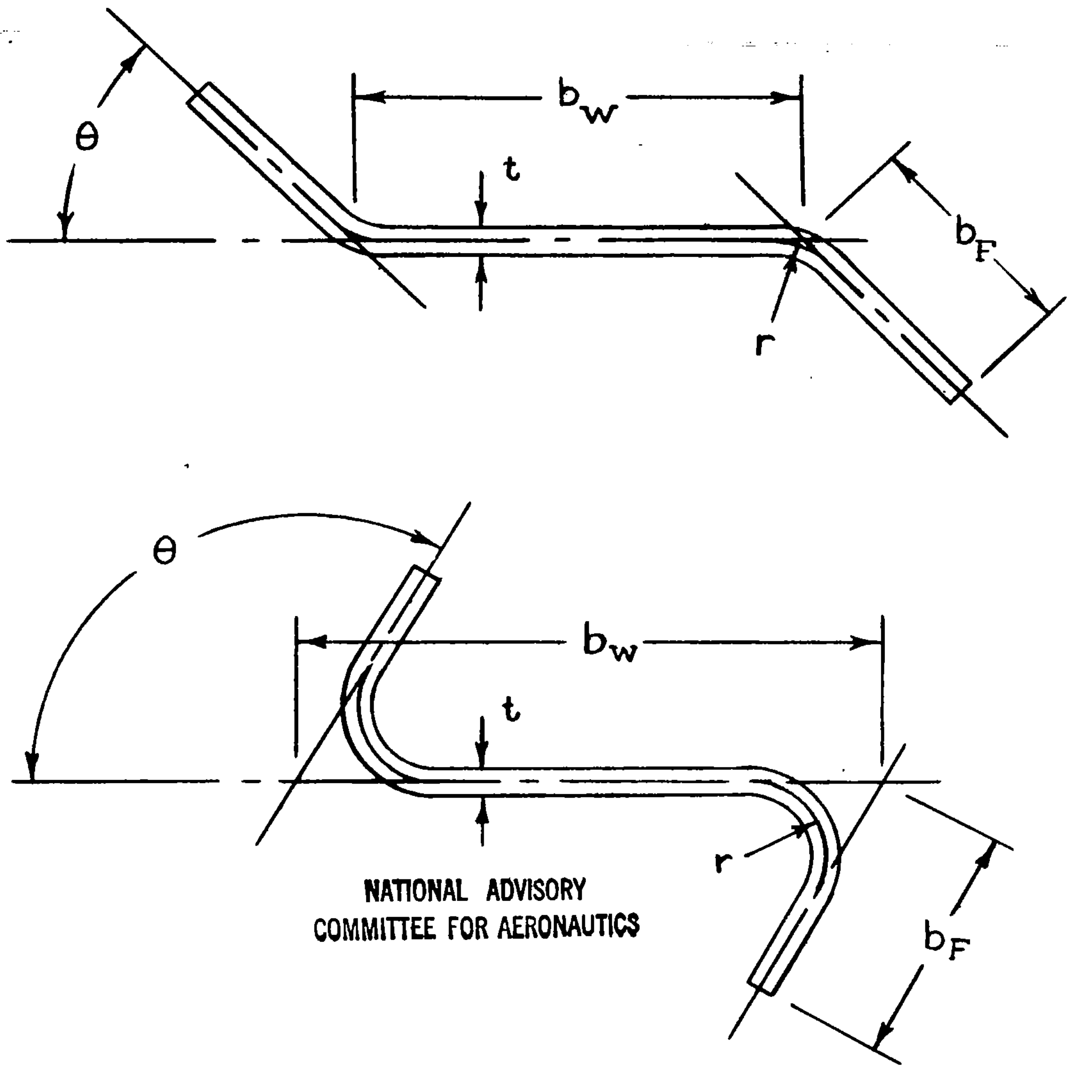

Figure 2.- Measured dimensions of cross section of test specimens.

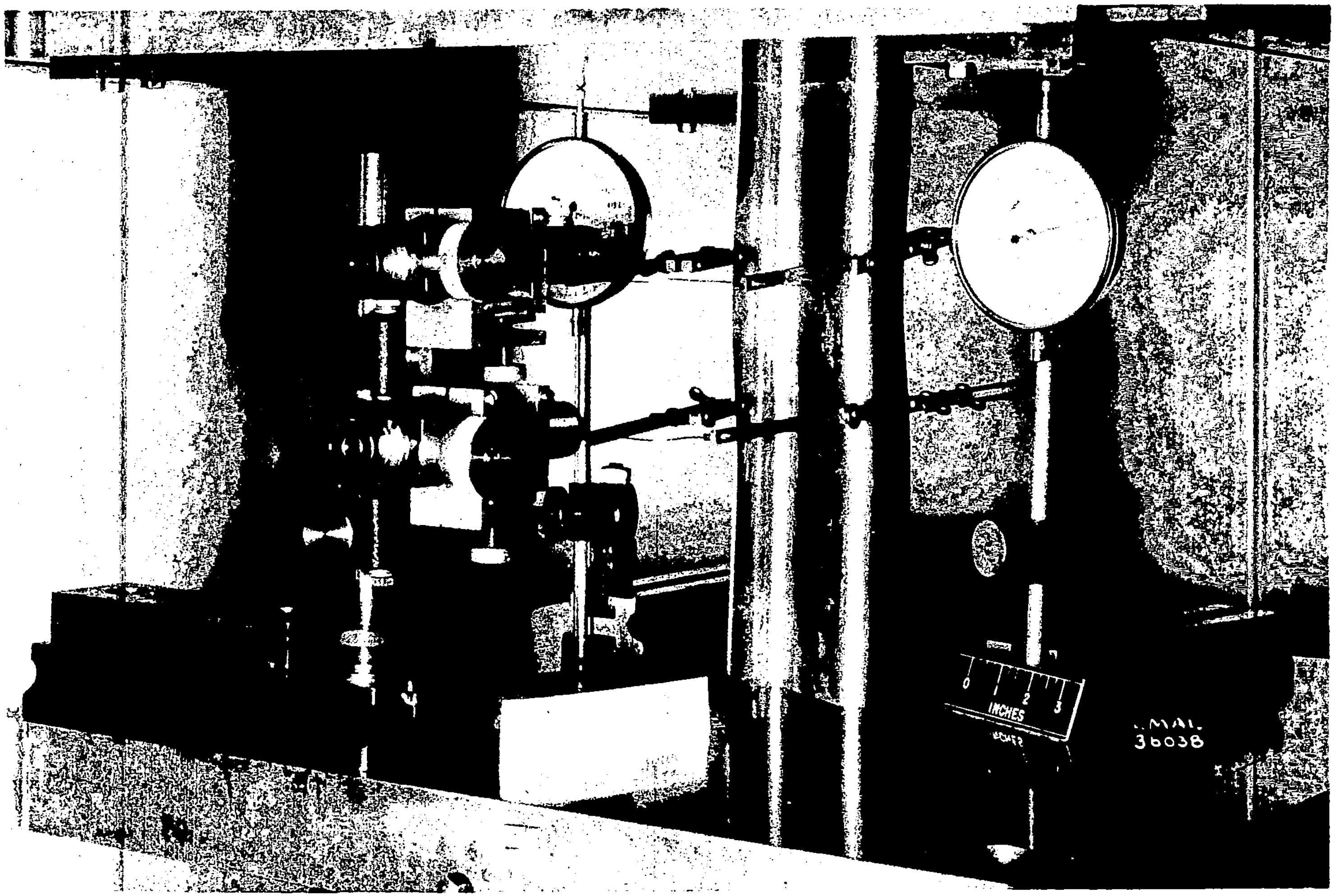

Figure 3.- Specimen of Z-section column under load.

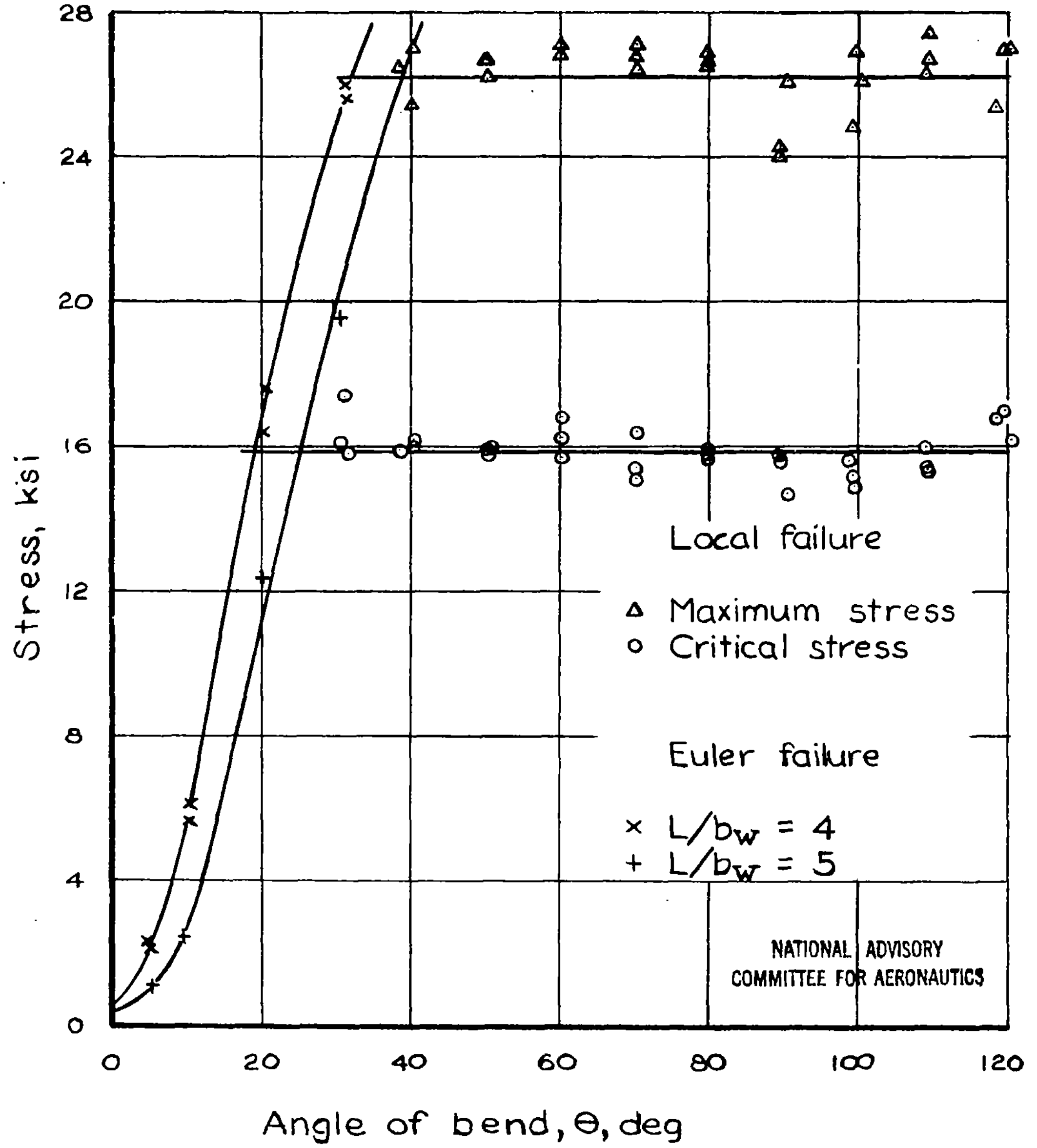

Figure 4.— Effect of angle of bend on the critical and maximum stress of 24 S-T aluminum-alloy columns of Z-section.

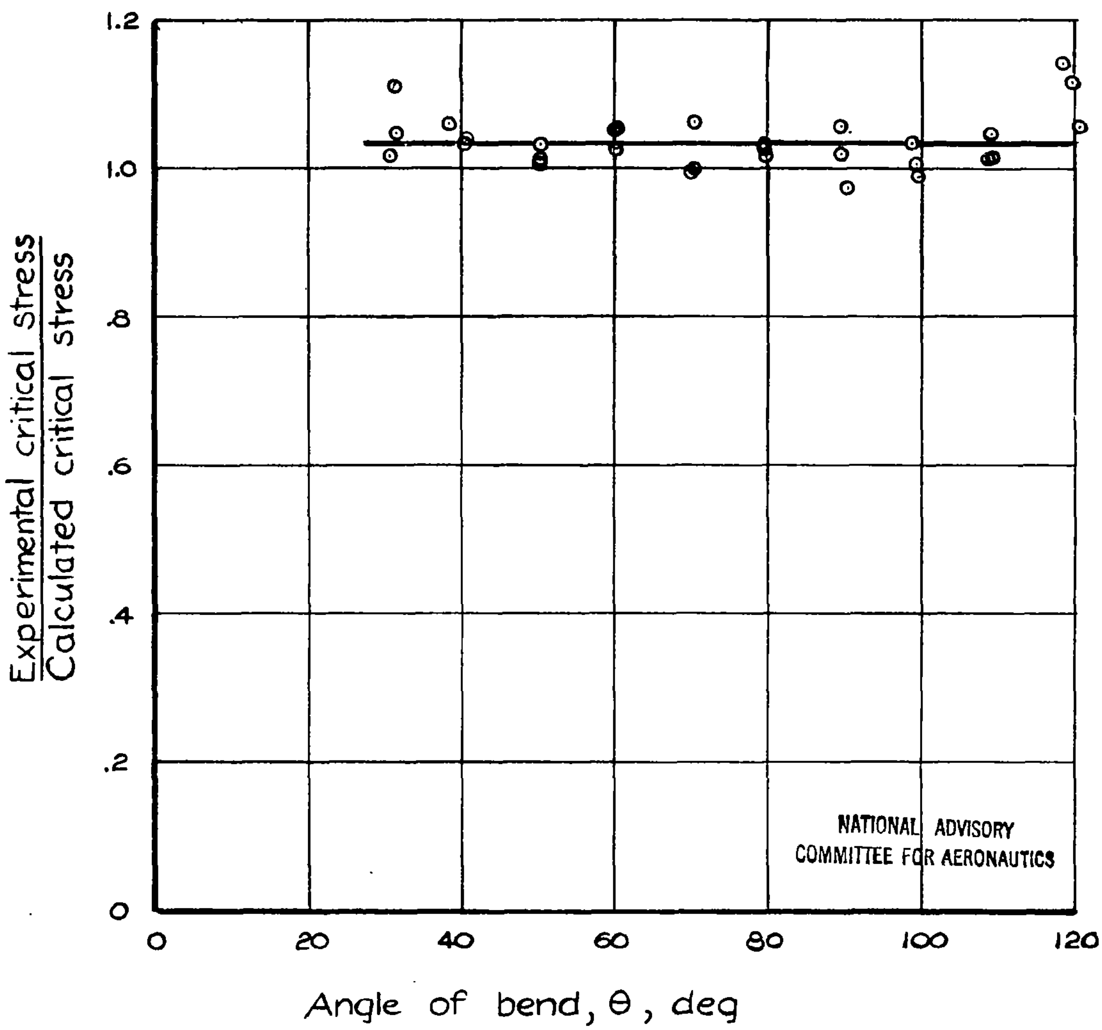

Figure 5. – Ratio of experimental to calculated critical stress for angles of bend.

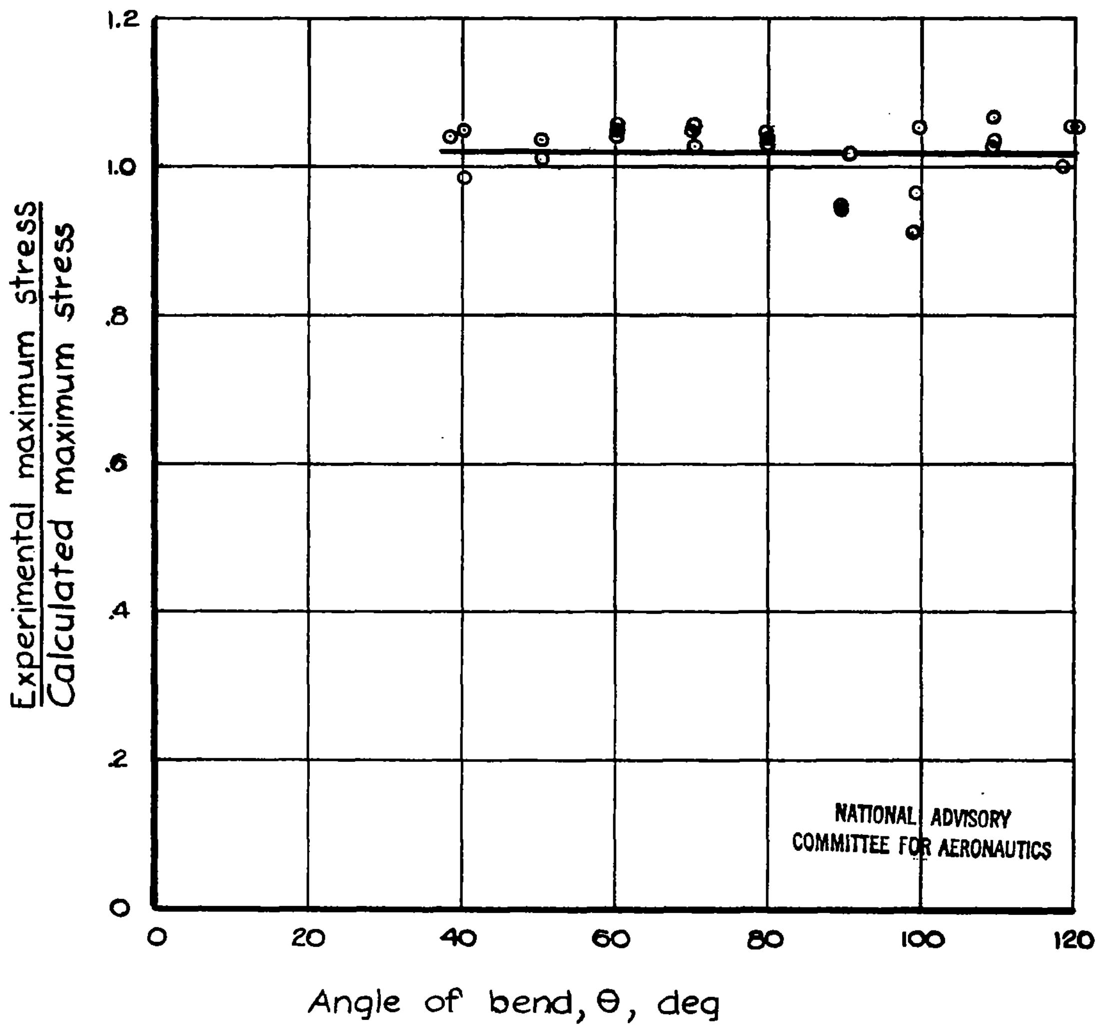

Figure 6.— Ratio of experimental to calculated maximum stress for angles of bend.

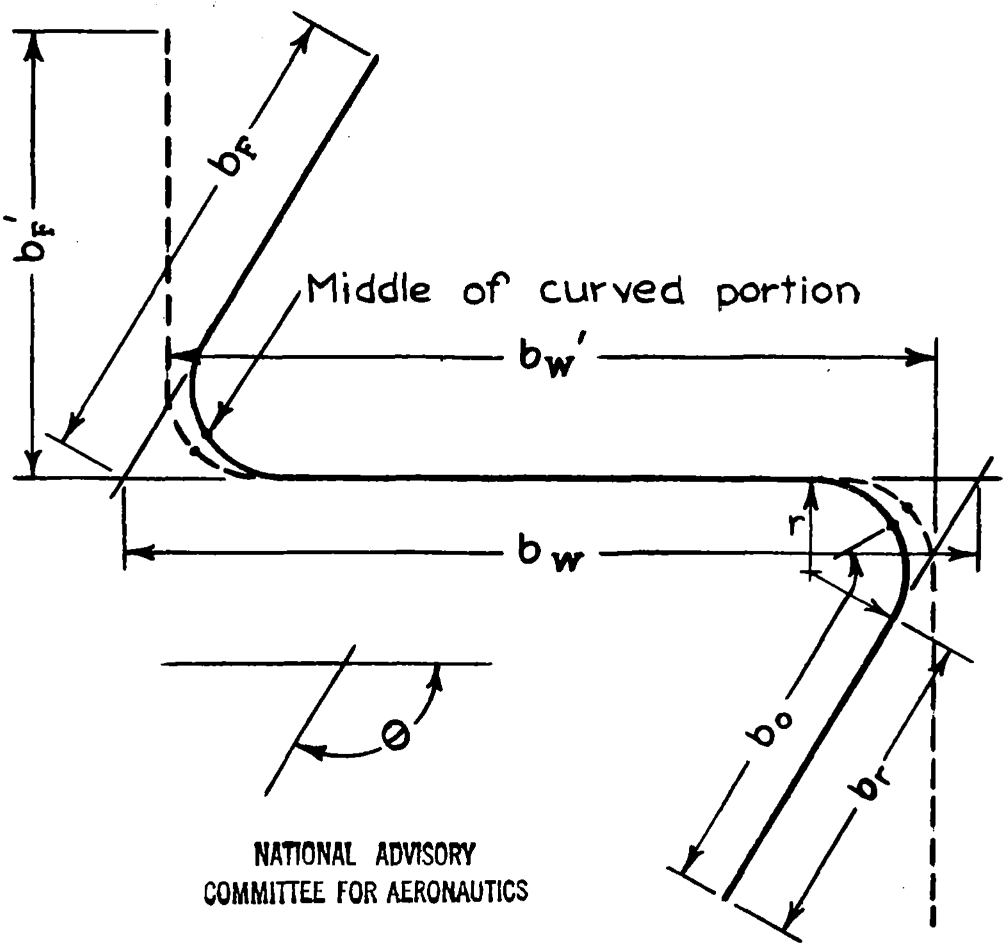

Figure 7. - Median-line diagram of specimen cross section showing adjustment to 90° bend.